Does Carbon Trading improve sustainability?

Let us now consider a situation involving production externalities. Firm S produces some amount of steel, *s*, and also produces a certain amount of pollution, *x*, which it dumps into a river. Firm F, a fishery, is located downstream and is adversely affected by S's pollution.

Suppose that firm S's cost function is given by $C_S(s, x)$, where *s* is the amount of steel produced and *x* is the amount of pollution produced.

Firm F's cost function is given by $C_f(f, x)$, where *f* indicates the production of fish and *x* is the amount of pollution.

Note that F's costs of producing a given amount of fish depend on the amount of pollution produced by the steel firm. We will suppose that pollution increases the cost of providing fish $\Delta C_f / \Delta x > 0$, and that pollution d*ecreases* the cost of steel production, $\Delta C_S / \Delta x \leq 0$. This last assumption says that increasing the amount of pollution will decrease the cost of producing steel—that reducing pollution will increase the cost of steel production, at least over some range.

The steel firm's profit-maximization problem is: $\max_{s,x}\ p_s s - C_s(s,x)$

and the fishery's profit-maximization problem is: $\max_{f}\ p_f f - C_f(f,x)$

Note that the steel mill gets to choose the amount of pollution that it generates, but the fishery must take the level of pollution as outside of its control.

The conditions characterizing profit maximization will be for steel plant:

$$p_s = \frac{\Delta C_s(s^*, x^*)}{\Delta s} \quad \text{and } 0 = \frac{\Delta C_s(s^*, x^*)}{\Delta x}$$

for the fishery:

$$p_f = \frac{\Delta C_f(f^*, x^*)}{\Delta f}$$

These conditions say that at the profit-maximizing point, the price of each good-steel and pollution-should equal its marginal cost. In the case of the steel firm, one of its products is pollution, which, by assumption, has a zero price. So, the condition determining the profit maximizing supply of pollution says to produce pollution until the cost of an extra unit is zero.

Case-1: If there is only one firm who is producing steel as well as fish

If there is only one firm, then it will take the interactions between its different "divisions" into account when it chooses the profit-maximizing production plan. Before the merger, each firm had the right to produce whatever amount of steel or fish or pollution that it wanted, regardless of what the other firm did. After the merger, the combined firm has the right to control the production of both the steel mill and the fishery.

The merged firm's profit-maximization problem is $\max_{s,f,x} \; p_s s + p_f f - C_s(s,x) - C_f(f,x)$

Which yields optimality conditions of:

$$p_s = \frac{\Delta C_s(s^*,x^*)}{\Delta s}$$

$$p_f = \frac{\Delta C_f(f^*,x^*)}{\Delta f}$$

$$0 = \frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} + \frac{\Delta C_f(\hat{f},\hat{x})}{\Delta x}$$

What does this imply about the amount of pollution produced? When the steel firm acted dependently, the amount of pollution was determined by the condition

$$\frac{\Delta C_s(s^*,x^*)}{\Delta x} = 0$$

In the merged firm, the amount of pollution is determined by the condition

$$\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} + \frac{\Delta C_f(\hat{f},\hat{x})}{\Delta x} = 0 \quad \text{----------------------- (1)}$$

or

$$MC_s(\hat{s},\hat{x}) + MC_f(\hat{f},\hat{x}) = 0$$

or

$$-\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} = \frac{\Delta C_s(\hat{f},\hat{x})}{\Delta x} > 0$$

That is, the merged firm produces pollution until the *sum* of the marginal cost to the steel mill and the marginal cost to the fishery is zero.

In this latter expression $\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x}$ is positive, since more pollution increases the cost of producing a given amount of fish. Hence the merged firm will want to produce where $-\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x}$ is positive; that is, it will it will want to produce *less* pollution than the independent steel firm.

Case-2: If there is Pollution Vouchers

Suppose that there are only two firms. Firm 1's emission quota is x_1 and firm 2's is x_2. The cost of achieving an emission quota x_1 is $C_1(x_1)$ and similarly for firm 2 it is $C_2(x_2)$. The total amount of emission is fixed at some target level X. If we want to minimize the total costs of achieving the emissions target, subject to the aggregate constraint, we need to solve the following problem:

$$\min_{x_1,x_2} C_1(x_1)+C_2(x_2)$$

such that

$$x_1+x_2=X$$

Let us rearrange the optimize problem as:

$$\max_{s,x_s} p_s s - C_1(x_s) - C_s(s,x)$$

$$\max_{f,x_f} p_f f - C_2(x_f) - C_f(f,x)$$

$$\min_{x_s,x_f} C_1(x_s)+C_2(x_f)$$

Such that:

$$x_s+x_f=X$$

Optimize problem will be:

$$p_s - \frac{\Delta C_s(s,x)}{\Delta s} = 0$$

$$-\frac{\Delta C_1(x_s)}{\Delta x_s} - \frac{\Delta C_s(s,x_s)}{\Delta x_s} = 0$$

$$p_f - \frac{\Delta C_f(f,x)}{\Delta f} = 0$$

$$-\frac{\Delta C_2(x_f)}{\Delta x_f} - \frac{\Delta C_f(f,x_f)}{\Delta x_f} = 0$$

and

$$\frac{\Delta C_1(x_s)}{\Delta x_s}+\frac{\Delta C_2(x_f)}{\Delta x_f}.\frac{\Delta x_f}{\Delta x_s}=0$$

$$\frac{\Delta C_1(x_s)}{\Delta x_s}.\frac{\Delta x_s}{\Delta x_f}+\frac{\Delta C_2(x_f)}{\Delta x_f}=0$$

$$\frac{\Delta x_f}{\Delta x_s}=-1$$

that is

$$\frac{\Delta C_1(x_s)}{\Delta x_s}=\frac{\Delta C_2(x_f)}{\Delta x_f}$$

Solving, we get:

$$p_s=\frac{\Delta C_s(\hat{s},\hat{x}_s)}{\Delta s}$$

$$p_f=\frac{\Delta C_f(\hat{f},\hat{x}_f)}{\Delta f}$$

Find out:

$$\frac{\Delta C_1(\hat{x}_s)}{\Delta x_s}+\frac{\Delta C_2(\hat{x}_f)}{\Delta x_f}.\frac{\Delta \hat{x}_f}{\Delta \hat{x}_s}$$

$$=\left[-\frac{\Delta C_s(\hat{s},\hat{x}_s)}{\Delta x_s}\right]+\left[-\frac{\Delta C_f(\hat{f},\hat{x}_f)}{\Delta x_f}\right]\left[-1\right]=0$$

or

$$\frac{\Delta C_s(\hat{s},\hat{x}_s)}{\Delta x_s}+\frac{\Delta C_f(\hat{f},\hat{x}_f)}{\Delta x_f}=0 \quad \text{------------------------} \quad (2)$$

A by now standard economic argument shows that the marginal cost of emission control must be equalized across the firms. If one firm had a higher marginal cost of emission control than the other, then we could lower total costs by reducing its quota and increasing the quota of the other firm.

How can we achieve this outcome? If the government regulators had information on the cost of emissions for all firms, they could calculate the appropriate pattern of production and impose it on all the relevant parties. But the cost of gathering all this information, and keeping it up-to-date, is staggering.

Case-3: If there is Pigouvian tax

One way to control the emission is to place a tax on the pollution generated by the steel firm. Suppose that we put a tax of t dollars per unit of pollution generated by the steel firm. Then the profit maximization problem of the steel firm becomes.

$$\max_{s,f,x} \; p_s s - C_s(s,x) - tx$$

Hence,

$$p_s - \frac{\Delta C_s(s,x)}{\Delta s} = 0$$

$$-\frac{\Delta C_s(s,x)}{\Delta x} - t = 0$$

If we put,

$$t = \frac{\Delta C_f(\hat{f},\hat{x})}{\Delta x}$$

we get $\dfrac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} + \dfrac{\Delta C_s(\hat{f},\hat{x})}{\Delta x} = 0$ --------------------------- (3)

Case-4: If fishery had the right to clean water

We could imagine a world where the fishery had the right to clean water, but could sell the right to allow pollution. Let q be the price per unit of pollution, and let x be the amount of pollution that the steel mill produces. Then the steel mill's profit-maximization problem is :

$$\max_{s,x} \; p_s s - qx - C_s(s,x)$$

and the fishery's profit-maximization problem is:

$$\max_{f,x} \; p_f f + qx - C_f(f,x)$$

$$p_s = \frac{\Delta C_s(s,x)}{\Delta s}$$
$$q = -\frac{\Delta C_s(s,x)}{\Delta x}$$
$$p_f = \frac{\Delta C_f(f,x)}{\Delta f}$$
$$q = \frac{\Delta C_f(f,x)}{\Delta x}$$

Thus each firm is facing the social marginal cost of each of its actions when it chooses how much pollution to buy or sell. If the price of pollution is adjusted until the demand for pollution equals the supply of pollution, we will have an efficient equilibrium, just as with any other good. The optimal solution, equations imply that

$$\hat{q} = -\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} = \frac{\Delta C_f(\hat{f},\hat{x})}{\Delta x}$$
$$\frac{\Delta C_s(\hat{s},\hat{x})}{\Delta x} + \frac{\Delta C_f(\hat{f},\hat{x})}{\Delta x} = 0 \text{ ------------------------------ (4)}$$

Conclusion

We could conclude that pollution emitted by the steel firm could be controlled any of the three ways:

1. Permitting the pollution emitter to extend its business to the area where it is effected by polluting.
2. Issuing the Pollution Vouchers or quota for each business.
3. Imposing Tax on Pollution emission.
4. One firm has a right to clean the emission and could sell the right to allow pollution.

In each case of the above 4 cases, the optimum emission equation number (2) to (4) is same as equation number (1).

Hence, we could presume that Carbon trading should optimize emotion since it provides to sell the right to allow pollution.

I here explain the carbon trading strategy in the light of Externality taken from in the book: Microeconomics, A modern approach by Hal R Varian